Comment on "Adaptive-feedback control algorithm"

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We point out a problem in the parametric estimation method based on the adaptive-feedback algorithm proposed by D. Huang [Phys. Rev. E **73**, 066204 (2006)] both illustratively and analytically. Furthermore, under some hypotheses, we prove that it is not the so-called chaotic dynamical characteristic but the linear independence between the right-hand functions and the estimated parameters of the intrinsic system in its attractor that counts in the availability of this algorithm in estimating the unknown parameters.

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In a recent paper [1], the author presented the detailed mathematical proof and remarks on his previous works [2–5], where an adaptive-feedback algorithm was proposed to stabilize and synchronize chaotic system. This adaptive-feedback algorithm can be briefly described as follows. Consider an n-dimensional system in the form of

$$\dot{x} = F(x,p), \tag{1}$$

where $x=(x_1,\ldots,x_n)^{\mathsf{T}} \in \mathbb{R}^n$ and $F(x,p)=[F_1(x,p),\ldots,F_n(x,p)]^{\mathsf{T}} \in \mathbb{R}^n$ is a nonlinear vector function with $F_i(x,p)=c_i(x)+\Sigma_{j=1}^m p_{ij} f_{ij}(x)$, $i=1,\ldots,n$. Here, $c_i(x)$, $f_{ij}(x)$, $i=1,\ldots,n, j=1,\ldots,m$, are nonlinear functions and $p=\{p_{ij}, i=1,\ldots,n, j=1,\ldots,m\} \in \mathbb{R}^{nm}$ are nm unknown parameters to be estimated. Also, uniform Lipschitz conditions are assumed to be satisfied, i.e., $|F_i(x,p)-F_i(y,p)| \leqslant l \max_j |x_j - y_j|$ holds for some $l>0$, all $x,y \in \mathbb{R}^n$, and $i=1,\ldots,n$. To estimate the unknown parameters p, the following receiver system was also introduced:

$$\dot{y} = F(y,q) - K(y-x), \tag{2}$$

where $K=\mathrm{diag}\{k_1,k_2,\ldots,k_n\}$, with the adaptive feedbacks:

$$\dot{q}_{ij} = -\delta_{ij} e_i f_{ij}(y),$$

$$\dot{k}_i = \gamma_i (x_i - y_i)^2, \tag{3}$$

where $\delta_{ij}>0$, $\gamma_i>0$ are some constants and $e_i=y_i-x_i$. In particular, theorem 4 in [1] indicated that the adaptive-feedback algorithm, Eqs. (1)–(3) can efficiently estimate the unknown parameter in the intrinsic system (1).

However, theorem 4 of [1] is problematic and its proof is not rigorous in mathematics. First, we present a counterexample for which this algorithm fails. Consider the following six-dimensional system:

$$\dot{x}_1 = a[x_2 - x_1 - h(x_1)],$$

$$\dot{x}_2 = x_1 - x_2 + x_3,$$

$$\dot{x}_3 = -bx_2,$$

$$\dot{x}_4 = a[x_5 - x_4 - h(x_4)] + p_1(x_1 - x_4),$$

$$\dot{x}_5 = x_4 - x_5 + x_6 + p_2(x_2 - x_5),$$

$$\dot{x}_6 = -bx_5 + p_3(x_3 - x_6), \tag{4}$$

where $a=9.78$, $b=14.97$, $h(x_1)=m_1x_1+0.5(m_0-m_1)[|x_1+1| - |x_1-1|]$ with $m_1=-0.75$ and $m_0=-1.31$, and $p_{1,2,3}$ are unknown parameters. This system can actually be regarded as coupled Chua's circuits, which was studied in [6]. The subsystem (x_1,x_2,x_3) is a Chua's circuit and the subsystem (x_3,x_4,x_5) is the another Chua's circuit coupled with the subsystem (x_1,x_2,x_3) by linear feedback. As shown in [6], (x_1,x_2,x_3) has a double-scrolling chaotic attractor.

The task is to estimate the unknown parameters $p_{1,2,3}$ by applying the adaptive-feedback algorithm proposed in [1], namely, Eqs. (1)–(3). To do so, we introduce the following receiver system:

$$\dot{y}_1 = a[y_2 - y_1 - h(y_1)] + k_1(x_1 - y_1),$$

$$\dot{y}_2 = y_1 - y_2 + y_3 + k_2(x_2 - y_2),$$

$$\dot{y}_3 = -by_2 + k_3(x_3 - y_3),$$

$$\dot{y}_4 = a[y_5 - y_4 - h(y_4)] + q_1(y_1 - y_4) + k_4(x_4 - y_4),$$

$$\dot{y}_5 = y_4 - y_5 + y_6 + q_2(y_2 - y_5) + k_5(x_5 - y_5),$$

$$\dot{y}_6 = -by_5 + q_3(y_3 - y_6) + k_6(x_6 - y_6), \tag{5}$$

with the following feedback:

$$\dot{q}_j = (y_j - y_{j+3})(x_{j+3} - y_{j+3}), \quad j=1,2,3,$$

$$\dot{k}_i = (x_i - y_i)^2, \quad i=1,\ldots,6. \tag{6}$$

Pick $p_1=p_2=p_3=10$. Let $\mathrm{var}_1=\langle(1/3)(|x_1-x_4|+|x_2-x_5|+|x_3-x_6|)\rangle$ denote the variance between two subsystems in the intrinsic system (4) and $\mathrm{var}_2=\langle(1/6)\Sigma_{i=1}^6|x_i-y_i|\rangle$ denote the variance between the intrinsic system (4) and the receiver system (5), where $\langle\cdot\rangle$ denotes the time average. Figure 1 shows that the variances $\mathrm{var}_{1,2}$ both converge to zero through time. However, even though the parameters $q_{1,2,3}$ converge, the terminal convergent values are not the estimated parameters $p_{1,2,3}$ and depend on the initial data. Therefore, from this synthetic example, the adaptive-feedback algorithm fails

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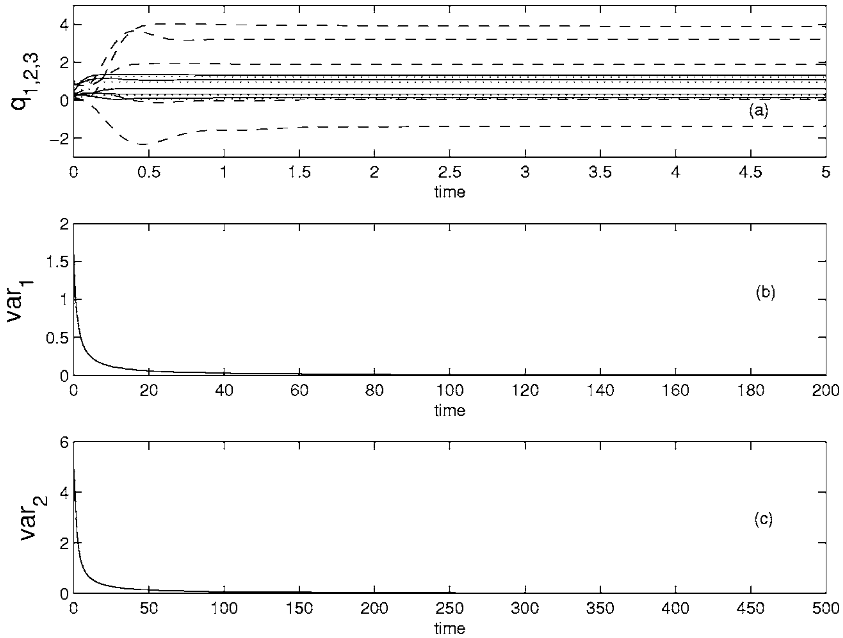

FIG. 1. (a) The variation of q_1 (solid line), q_2 (dotted line), and q_3 (dashed line) for overlapping five times with different randomly selected initial data; (b) the convergence of var_1 with respect to time; (c) the convergence of var_2 with respect to time.

to estimate the unknown parameter of the intrinsic system even though it does synchronize the receiver system to the intrinsic system.

The problem of the conclusion of theorem 4 of [1] lies on the misusage of the Lasalle invariance principle [7] in its proof of theorem 4. In this proof, the author constructed a scalar function:

$$V(x,y,q,k) = \frac{1}{2}\sum_{i=1}^{n} e_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\delta_{ij}}(q_{ij}-p_{ij})^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\gamma_i}(k_i-L)^2,$$

where $L > nl$. Differentiating it along the systems (1)–(3) gives

$$\dot{V}(x,y,q,k) \leqslant (nl-L)\sum_{i=1}^{n} e_i^2. \tag{7}$$

According to the Lasalle invariance principle, this implies that the flow (x,y,q,k) will converge to the largest invariant set contained in the set $E=\{(x,y,q,k):\dot{V}=0\}=\{(x,y,q,k):x=y\}$. Then the author claimed that the set $M=\{(x,y,q,k):x=y,q=p,k=k_0\}$ is the largest invariant set in E. But, this claim is *incorrect*. Consider the example we proposed above. Picking $p_1=p_2=p_3=10$ guarantees that the subsystem (x_1,x_2,x_3) is synchronized to the subsystem (x_4,x_5,x_6), namely, the synchronization manifold

$$S = \{(x_1,\ldots,x_6): x_1 = x_4, x_2 = x_5, x_3 = x_6\}$$

is an asymptotically stable invariant manifold for the system (4). This implies that the following set $M'=\{(x,y,q,k),x=y,q\in\mathbb{R}^3,k\in\mathbb{R}^6\}$ is also an invariant set contained in E through the evolution (4)–(6), where $x=(x_1,\ldots,x_6)^{\mathsf{T}}$, $y=(y_1,\ldots,y_6)^{\mathsf{T}}$, $q=(q_1,q_2,q_3)^{\mathsf{T}}$, and $k=(k_1,\ldots,k_6)^{\mathsf{T}}$. Obviously, we can see that M' nontrivially contains M, which implies that, in this example, M is not the largest invariant set contained in E. Therefore, the proof of theorem 4 in [1] is incorrect and its conclusion that the parameters in the receiver system can converge to the unknown parameter of the intrinsic system is problematic. Also, remark 5 in [1] as well as the proof claimed that this availability of estimating parameters relied on the "chaotic characteristic of system." However, in our example, the intrinsic system (4) has chaotic dynamics but its parameters fail to be estimated.

In fact, it is the characteristic of the attractor of the intrinsic system related to the unknown parameters that counts in the estimation of parameters. Hereby, we will present the necessary and sufficient condition for succeeding in estimating the parameters. Before that, we introduce the concept of attractor for a dynamical system in the Milnor sense. For more details, we refer the interested readers to [8]. Here, for the system (1), for each $x_0\in\mathbb{R}^n$, $\omega(x_0)$ denotes the ω-limit set of the orbit $x(t)$ with $x(0)=x_0$. Let $\rho(A)$ denote the set consisting of all points $x\in\mathbb{R}^n$ for which $\omega(x)=A$. If A is minimal, then we can conclude that $\rho(A)$ precisely equals to $B(A)$ up to a set with zero measure [8] and also has positive measure. Then, we can easily have the following conclusion.

Proposition 1. Suppose that all conditions in theorem 4 [1] *are satisfied. Let A be a minimal attractor of the intrinsic system* (1) *and suppose that the parameter orbits $q(t)$ and $k(t)$ converge* [9–11]. *If for any initial data $x_0\in\rho(A)$ and $(y_0,q_0,k_0)\in\mathbb{R}^{mn+2n}$, the receiver system and adaptive-feedback system can precisely estimate the unknown parameter p, namely,* $\lim_{t\to\infty}q(t)=p$, *if and only if $\sum_{i,j}p_{ij}f_{ij}(x)=0$ holding in the attractor A implies $p_{ij}=0$ for all $i=1,\ldots,n$, $j=1,\ldots,m$.*

In the case that the attractor A is not minimal, A can be decomposed into at most countable disjoint minimal attractors $\{A_j\}_{j=1}^{l}$, where l can be ∞. Then, the availability of the estimation of the parameters can be guaranteed if the functions $\{\tilde{f}_{ij}(x)\}$ are linearly independent in each minimal attractor A_j, $j=1,\ldots,l$.

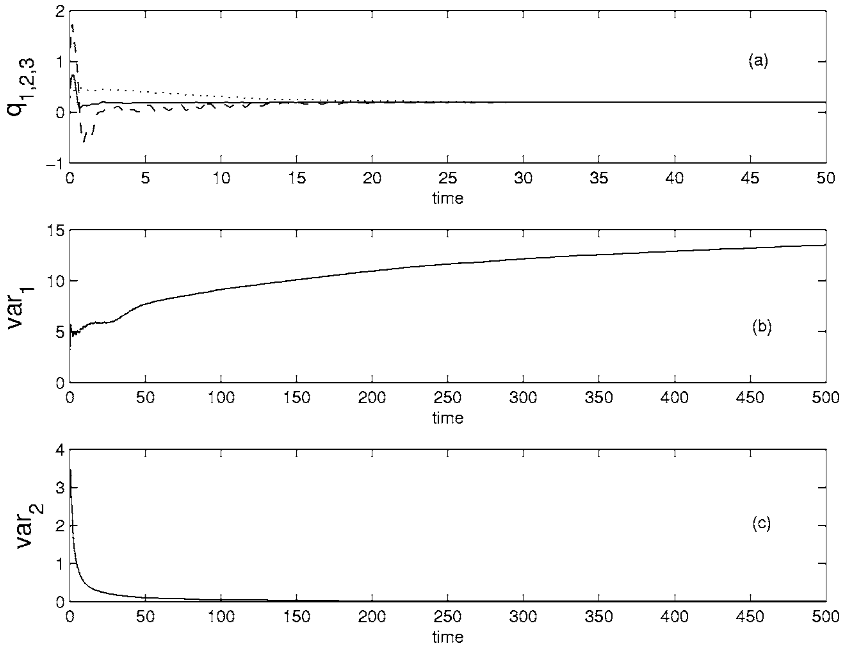

FIG. 2. (a) The variation of q_1 (solid line), q_2 (dotted line), and q_3 (dashed line) with randomly selected initial data; (b) the variance of var_1 with respect to time; (c) the convergence of var_2 with respect to time.

Coming back to the example presented above, with $p_1=p_2=p_3=10$, one can see that the synchronization manifold S is asymptotically stable, which implies any attractor of the system (4) is contained in S. The vector functions corresponding to the functions $\{x_i-x_{i+3}\}_{i=1}^3$ is obviously linear dependent in S since $x_i-x_{i+3}=0$ holds for all $x\in S$ and $i=1,2,3$. By proposition 1, we can see that in the case the parameters $p_{1,2,3}$ cannot be estimated by $q_{1,2,3}$, respectively. This is also shown in Fig. 1.

However, picking $p_1=p_2=p_3=0.2$, which cannot synchronize the subsystem (x_4,x_5,x_6) to the subsystem (x_1,x_2,x_3), applying the receiver and adaptive-feedback systems (5) and (6), Fig. 2 shows that the orbits $q_{1,2,3}(t)$ all converge to 0.2 but var_1 does not converge to zero, which implies that the subsystem (x_4,x_5,x_6) is not synchronized to the subsystem (x_1,x_2,x_3). Moreover, var_2 converges to zero which implies the receiver system (5) is synchronized to the intrinsic system (4) via this adaptive-feedback algorithm (6).

In conclusion, the adaptive-feedback algorithm proposed in [1] could not surely estimate the unknown parameters in the intrinsic system. The availability of this algorithm can be guaranteed by the linear independence of the functions, which are linearly involved by these parameters, in the attractor of the intrinsic system. Therefore, in a rigorous mathematical viewpoint, without sufficient knowledge of the attractor of the intrinsic system, the availability of this estimation algorithm cannot be guaranteed.

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[11] From the derivative of the scalar function V, Eq. (7), one can conclude that $\int_0^\infty \Sigma_{i=1}^n e_i^2(\tau)d\tau \leqslant [1/(L-nl)]V(x(0),y(0),q(0),k(0)) < +\infty$, namely, $\Sigma_{i=1}^n e_i^2(t)\in L^2([0,+\infty))$. From the adaptive feedback (3), this implies that the orbit $k(t)$ is a Cauchy series. In other words, the convergence of $k(t)$ can be concluded. However, the convergence of $q(t)$ cannot be proved to the best of my knowledge and reasoning. Furthermore, in [9,10], the author provided the so-called "persistently exciting" condition as a sufficient condition for the convergence of the parameter trajectory $q(t)$. One can see that this "persistently exciting" condition actually implies the condition in proposition 1.